Critical Trap Aspect Ratios For Dipolar BEC

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We show that there exists critical trap aspect ratios for a trapped Bose-Einstein condensate with dipole-dipole interactions. We discuss the role of critical trap aspect ratios on both the critical angular velocity above which a vortex is energetically favorable and the precession velocity of an off-axis vortex in TF regime. We show that the stability diagram for a purely dipolar gas depends crucially on the critical trap aspect ratios both for positive and negative strength of dipolar interaction. We prove that the critical points can be changed by an anharmonic potential.

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I. INTRODUCTION

The ultracold gases with dipole-dipole interaction became the subject of intensive theoretical studies after the first gaseous condensate was produced. The experimental realization of Bose-Einstein condensation of ^{52}Cr atoms in 2005 has raised the interest in study of BEC with nonlocal dipole-dipole interactions [1–4]. This nonlocal character of the potential has remarkable consequences for the physics of rotating dipolar gases in TF limit. It is believed that, in axially symmetric traps with the axis along the dipole orientation, the critical angular velocity, above which a vortex is energetically favorable, is decreased due to the dipolar interaction in oblate traps, and increased in prolate traps [5–7]. It was noted in [8] that the effect of the dipole-dipole interaction is the lowered (raised) precession velocity of an off-center straight vortex line in an oblate (a prolate) trap.

The nonlocal character of dipolar potential is also crucial for the stability properties of dipolar gases. It was shown that there exists a critical aspect ratio in the stability diagram of a purely dipolar condensate [9].

In this paper, we show that the crossover between a reduced and an increased critical angular velocity above which a vortex is energetically favorable occurs at a critical aspect ratio in TF regime. We discuss the role of the critical aspect ratio on the precession velocity of an off-center straight vortex in TF limit. In the Gaussian limit, we show that there exists two critical aspect ratios for a purely dipolar condensate. We investigate their role on the stability diagram. We discuss that an additional anharmonic potential changes the critical trap aspect ratios.

The paper is organized as follows: The following section reviews two main methods to calculate the dipolar potential interaction. Section III finds a critical trap aspect ratio in TF limit. Section IV investigates critical trap aspect ratio in the stability diagram and discusses the effect of additional quartic potential on the critical aspect ratios.

II. DIPOLAR INTERACTION

Consider a BEC of N particles with mass m and magnetic dipole moment μ oriented in the z-direction by a sufficiently large external field. At sufficiently low temperatures, the description of the ground state of the condensate is provided by the solution of the Gross-Pitaevskii (GP) equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_T + g|\Psi|^2 + \Phi_{dd}\right)\Psi = \mu\Psi \ , \qquad (1)$$

where $\Phi_{dd}(\mathbf{r}) = \frac{C_{dd}}{4\pi}\int d^3\mathbf{r}'\frac{1-3\cos^2\theta}{|\mathbf{r}-\mathbf{r}'|^3}|\Psi(\mathbf{r}',t)|^2$ is the mean field term due to dipole-dipole interactions, C_{dd} is the strength of the dipolar interactions, θ is the angle between the vector connecting two dipolar particles and the dipole orientation, $g = \frac{4\pi\hbar^2 a_s}{m}$, a_s is the scattering length, V_T is the trap potential

$$V_T(\mathbf{r}) = \frac{m}{2}\omega_\perp^2\left(r^2 + \ \gamma^2 z^2\right) \ , \qquad (2)$$

where $\gamma = \frac{\omega_z}{\omega_\perp}$ is the trap aspect ratio. It is well known that the sign of the dipolar mean-field energy can be controlled via the trap aspect ratio. For dipolar condensates, it is useful to introduce a dimensionless parameter that measures the relative strength of the dipolar and s-wave interactions $\varepsilon_{dd} = \frac{C_{dd}}{3g}$. Chromium atoms posses an anomalously large magnetic dipole moment, hence $\varepsilon = 0.16$. It can be enhanced via Feshbach tuning of the scattering length. In Thomas-Fermi regime, dipolar BEC is stable as long as $-0.5 < \varepsilon_{dd} < 1$.

The equation (1) is an integro-differential equation since it has both integrals and derivatives of an unknown wave function. Let us focus on the dipolar potential term, Φ_{dd}. There are two main methods to calculate Φ_{dd}. The first one is based on a Gaussian trial function. Φ_{dd} can be calculated by employing Fourier transform from (r, z)-space to $(k_\perp, k_z)$-space. The Fourier transform of the dipole-dipole interaction term in the small cut-off distance is given by $\mathcal{F}\left\{\frac{1-3\cos^2\theta}{|\mathbf{r}-\mathbf{r}'|^3}\right\} = \frac{4\pi}{3}\ (3\cos^2\alpha - 1)$

[10, 11]. Hence, the convolution theorem leads

$$\Phi_{dd} = \frac{C_{dd}}{3}\mathcal{F}^{-1}\left\{(3\cos^2\alpha - 1)\mathcal{F}\left\{|\Psi|^2\right\}\right\} \qquad (3)$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier and the inverse Fourier transforms, respectively and α is the angle between the momentum $\mathbf{k}$ and the dipole direction. Since we have assumed that the dipoles are polarized along the z-axis, $\cos^2\alpha = \frac{k_z^2}{k_z^2 + k_\perp^2}$. The corresponding dipole-dipole energy can be calculated using eq. (3) and the Parseval's theorem

$$E_{dd} = \frac{C_{dd}}{6}\int(3\cos^2\alpha - 1)\left|\mathcal{F}\{|\Psi|^2\}\right|^2 d^3\mathbf{k} \qquad (4)$$

where $d^3\mathbf{k} = 2\pi\ k_\perp dk_\perp\ dk_z$. The last relation is of importance in the variational method, which is an efficient method to study the stability properties and collective excitations of dipolar gas [9–19].

The second method to calculate Φ_{dd} was presented by Eberlein, Giovanazzi and O'Dell in TF regime [20]. They showed that a parabolic density profile remains an exact solution for an harmonically trapped vortex-free dipolar condensate in the TF limit. In the TF regime, the mean-field dipolar potential, $\Phi_{dd}(\mathbf{r})$, is given by

$$\Phi_{dd}(\rho, z) = \frac{n_0 C_{dd}}{3}\left(\frac{\rho^2}{R^2} - \frac{2z^2}{L^2} - f(\kappa)(1 - \frac{3}{2}\frac{\rho^2 - 2z^2}{R^2 - L^2})\right) \qquad (5)$$

where the function $f(\kappa)$ for oblate case, ($\kappa > 1$), is given by $f(\kappa) = \frac{2 + \kappa^2(4 - 6\ \frac{\arctan\sqrt{\kappa^2-1}}{\sqrt{\kappa^2-1}}\ )}{2(1-\kappa^2)}$. Here, $\kappa = \frac{R}{L}$ is the condensate aspect ratio. In the absence of dipolar interaction, the condensate aspect ratio and the trap aspect ratio match.

III. A SINGLE VORTEX

The density profile of a dipolar condensate with a straight central vortex line in the TF regime reads [5]

$$n(\mathbf{r}) = n_0\frac{\rho^2}{\rho^2 + \beta^2}\left(1 - \frac{\rho^2}{R^2} - \frac{z^2}{L^2}\right) \qquad (6)$$

where $n(\mathbf{r}) = 0$ when the right hand side is negative and β, R, and L are variational parameters that describe the size of the vortex core, the radial and the axial sizes, respectively. O'Dell and Eberlein derived the corresponding energy expression elegantly and performed numerical calculations for a prolate trap with $\gamma = 0.2$ and an oblate trap with $\gamma = 5$ [5]. They found that the critical angular velocity, Ω_c, above which a vortex state is energetically favorable, is increased for the former one and decreased for the latter one. Based on the results for $\gamma = 0.2$ and $\gamma = 5$, the authors concluded that Ω_c is decreased due to the dipole-dipole interaction in oblate traps, $\gamma > 1$, and increased in prolate traps, $\gamma < 1$. We find that this generalization does not hold. The crossover between a reduced and an increased critical velocity occurs at $\gamma = \gamma_c > 1$ rather than $\gamma = 1$ in TF regime. In other words, Ω_c are the same for dipolar and non-dipolar condensates at γ_c. The critical value γ_c changes slightly with ϵ_{dd}. For ^{52}Cr atoms with $\epsilon_{dd} = 0.16$, $\gamma_c \approx 2.8$. For $\varepsilon_{dd} = 0.6$, $\gamma_c \approx 2.9$. Fig-1 plots Ω_c in units of $\omega_\perp$ as a function of trap aspect ratio γ for three different values of ε_{dd} ($\varepsilon_{dd} = 0$, $\varepsilon_{dd} = 0.2$, and $\varepsilon_{dd} = 0.6$). As can be seen from the figure, Ω_c increases with ε_{dd} for $\gamma < \gamma_c$, while Ω_c decreases with ε_{dd} for $\gamma > \gamma_c$.

So far, we have focused on a central vortex. In [8], based

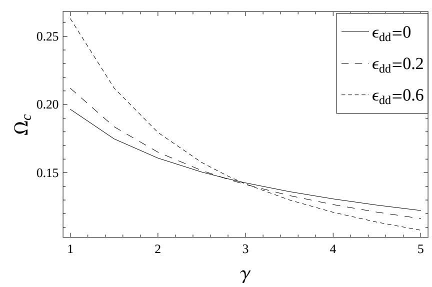

FIG. 1: The critical angular velocity of a condensate for $\varepsilon_{dd} = 0$, $\varepsilon_{dd} = 0.2$, and $\varepsilon_{dd} = 0.6$ as a function of trap aspect ratio γ. Ω_c is measured in units of $\omega_\perp$. Ω_c increases (decreases) with ε_{dd} for $\gamma < \gamma_c$ ($\gamma > \gamma_c$), where $\gamma_c \approx 2.8$

on two examples, it was concluded that the effect of the dipole-dipole interaction is the lowered (raised) precession velocity of an off-center straight vortex line in an oblate (a prolate) trap. We conclude that the effect of the dipole-dipole interaction is the lowered (raised) precession velocity of an off-center straight vortex line for $\gamma < \gamma_c$ ($\gamma > \gamma_c$).

Having found a critical trap aspect ratio in TF limit, let us search for the existence of a critical trap aspect ratio in Gaussian approximation.

IV. STABILITY

In this section, we will explore another critical value missed in earlier papers investigating stability properties of harmonically trapped dipolar condensates. Consider a pure dipolar condensate polarized in the z-direction. Note that due to the presence of a Feshbach resonance, a practically pure dipolar system can be produced by tuning the s-wave scattering length to zero.

A variational method can be employed to obtain an estimate for the instability conditions. We will use the notation used in [9].

$$\gamma \rightarrow \frac{1}{l^2}\ , \quad \kappa \rightarrow \frac{1}{\kappa}\ . \qquad (7)$$

We employ the following trial function

$$\Psi(\mathbf{r}) = \sqrt{\frac{N}{\pi^{3/2}\sigma^3 d_\perp^3 \kappa}}\ \exp\left(-\frac{1}{2\sigma^2 d_\perp^2}(r^2+\frac{z^2}{\kappa^2})\right)\ , \quad (8)$$

where $d_\perp = \sqrt{\hbar/m\omega_\perp}$ is the oscillator strength and the variational parameters σ measures the change in radial dimension and κ is the condensate aspect ratio.
The total energy $E^0 = E^0_{dd} + E^0_{nd}$, where E^0_{dd}, E^0_{nd} are dipolar and non-dipolar parts, is given by

$$\begin{aligned} \frac{E^0_{dd}}{E_\perp} &= -\sqrt{\frac{2}{\pi}}\frac{N^\star}{\sigma^3}\left(\frac{1}{3\kappa} - \frac{F(\kappa)-1}{\kappa(\kappa^2-1)}\right)\ , \\ \frac{E^0_{nd}}{E_\perp} &= (1+\frac{\kappa^2}{2l^4}\ )\sigma^2 + (1+\frac{1}{2\kappa^2}\ )\sigma^{-2}\ . \end{aligned} \quad (9)$$

where $E_\perp = \frac{1}{2}N\hbar\omega_\perp$, the function $F(\kappa) = \frac{\kappa\cosh^{-1}(\kappa)}{\sqrt{\kappa^2-1}}$ is a continuous function ($\kappa > 0$) and

$$N^\star = \frac{m\ C_{dd}}{4\pi\hbar^2 d_\perp}N\ . \quad (10)$$

We investigate the existence of minimum of the total energy as a function of the two variational parameters, σ and κ. The minimum conditions are given by $\frac{\partial E}{\partial\sigma} = 0$ and $\frac{\partial E}{\partial\kappa} = 0$. If we substitute the total energy, E^0, into these equations, we obtain

$$N^{\star 4} = \frac{\frac{324\pi^2(\kappa^2-1)^8 l^4}{(\kappa^4-l^4)^{-4}\kappa^6}\left((\frac{9}{\kappa^6}+\frac{18}{\kappa^2})F - 2 - \frac{18}{\kappa^2} - \frac{3}{\kappa^4} - \frac{4}{\kappa^6}\right)}{\left((\frac{9}{\kappa^4}+\frac{18l^4}{\kappa^4})F + 2 - \frac{7}{\kappa^2} - \frac{4}{\kappa^4} - \frac{2l^4}{\kappa^2}(1+\frac{10}{\kappa^2}-\frac{2}{\kappa^4})\right)^5} \quad (11)$$

$$\frac{\sigma}{N^\star} = \frac{(\frac{9}{\kappa^4}+\frac{2l^4}{\kappa^4})F + 2 - \frac{7}{\kappa^2} - \frac{4}{\kappa^4} - \frac{2l^4}{\kappa^2}(1+\frac{10}{\kappa^2}-\frac{2}{\kappa^4})}{3\sqrt{2\pi}\ \kappa(1-\frac{1}{\kappa^2})^2(1-\frac{l^4}{\kappa^4})} \quad (12)$$

The first one enables us to determine the condensate aspect ratio, κ, for a given number of particles and trap aspect ratio. The second equation gives the information how the radial and axial lengths change with κ and l.
To find the critical number of particles, we should also calculate the Hessian, $\Delta = \frac{\partial^2 E^0}{\partial\sigma^2}\frac{\partial^2 E^0}{\partial\kappa^2} - \left(\frac{\partial^2 E^0}{\partial\sigma\partial\kappa}\right)^2$. The local minimum of energy disappears and instability of the condensate occurs when the Hessian is equal to zero. The Hessian is too long to write here. Below, we will predict the existence of a critical trap aspect ratio.

A. Critical Points

There exists two critical values for the trap aspect ratio

$$l^\star = 0.43\ , \quad l^{\star\star} = 2.52\ . \quad (13)$$

Only the first critical point in Gaussian limit was predicted in the literature [9]. The critical trap aspect ratios play important roles in understanding of the general structure of dipolar condensates. Firstly, the dipole-dipole interaction is positive when $l < l^\star$ ($l > l^{\star\star}$) for $C_{dd} > 0$ ($C_{dd} < 0$). Hence, the dipolar condensate will be stable at any number of particles as long as $l < l^\star$ for $C_{dd} > 0$ and $l > l^{\star\star}$ for $C_{dd} < 0$. The dipole-dipole interaction is negative when $l > l^{\star\star}$ ($l < l^\star$) for $C_{dd} > 0$ ($C_{dd} < 0$). So, the stability requires $N^\star < N^\star_c$ where $N^\star_c$ is the critical a critical value. The critical number $N^\star_c$, contrary to the case of contact interaction, depends strongly on the trap geometry. Let us suppose $\kappa = \kappa_c$ and $\sigma = \sigma_c$ when $N^\star = N^\star_c$. It is interesting to observe that $N^\star_c \approx 6.2$, $\kappa_c \approx 2.6$, and $\sigma_c \approx 0.68$ remain almost the same when $l > 3$ for $C_{dd} > 0$. Fig-2 and Fig-3 plot $|N^\star_c|$ (solid curve) and κ_c (dashed curve) as a function of trap aspect ratio for $C_{dd} > 0$ and $C_{dd} < 0$, respectively. Amazingly, $N^\star_c$ and κ_c change sharply at $l = 0.8$ and $l = 0.7$, respectively for $C_{dd} > 0$ and at $l = 2.3$ and $l = 2.1$, respectively for $C_{dd} < 0$. As can be seen from the figures, there exists a minimum for critical number, $N^\star_c = 4.64$ at $l = 0.7$ for $C_{dd} > 0$, while no such minimum occurs for $C_{dd} < 0$. We note that in the region $l^\star < l < l^{\star\star}$, the sign of the dipole-dipole interaction depends on C_{dd} and l.
Secondly, the condensate aspect ratio is equal to the trap aspect ratio, $\kappa = l$, only if $l = l^\star$ or $l = l^{\star\star}$. Only in the region $l^\star < l < l^{\star\star}$, κ is bigger (smaller) than l for $C_{dd} > 0$ ($C_{dd} < 0$). As $N^\star$ is increased, κ increases (decreases) when $l^\star < l < l^{\star\star}$ ($l < l^\star$ or $l > l^{\star\star}$) for $C_{dd} > 0$. Conversely, as $N^\star$ is increased, κ decreases (increases) when $l^\star < l < l^{\star\star}$ ($l < l^\star$ or $l > l^{\star\star}$) for $C_{dd} < 0$. The dipole-dipole interaction stretches (squeezes) the cloud both in the radial and the axial directions compared to the non-dipolar condensate if $l < l^\star$ for $C_{dd} > 0$ ($C_{dd} < 0$) and if $l > l^{\star\star}$ for $C_{dd} < 0$ ($C_{dd} > 0$).
It is remarkable to note that the existence of two critical trap aspect ratios, 0.44 and 2.30 (in our notation), for a metastable TF solution outside of the regime $-0.5 < \varepsilon_{dd} < 1$ was discussed [22]. It is interesting to observe that these critical values in TF approximation are very close $l^\star$ and $l^{\star\star}$ in Gaussian approximation (13). It was shown that collapse via scaling oscillations is suppressed in TF limit if $l > 2.30$ for $\varepsilon_{dd} \to -\infty$ and if $l < 0.44$ for $\varepsilon_{dd} \to \infty$ [22].
In the following two subsections, we will show that the critical trap aspect ratios are changed by an additional anharmonic potential

$$V_T = \frac{m}{2}\omega_\perp^2\left(r^2 + \ \frac{z^2}{l^4} + \lambda_\perp\ \frac{r^4}{d_\perp^2} + \lambda_z\ \frac{z^4}{l^6 d_\perp^2}\right)\ , \quad (14)$$

where $\lambda_\perp$ and λ_z are small dimensionless parameters characterizing the strength of the quartic potential in radial and axial directions, respectively. This potential has been achieved experimentally by superimposing a blue detuned laser beam with the Gaussian profile to the magnetic trap [21].

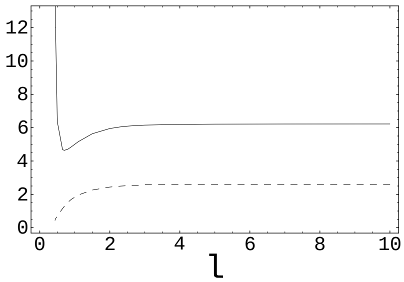

FIG. 2: The critical number of atoms, $N_c^\star$, (solid) and the critical value of the condensate aspect ratio, κ_c, (dotted) as a function of the trap aspect ratio, l, for a purely dipolar condensate with $C_{dd} > 0$.

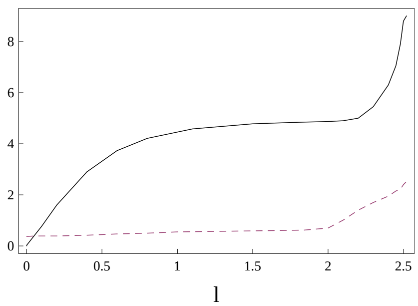

FIG. 3: The critical number of atoms, $|N_c^\star|$, (solid) and the critical value of the condensate aspect ratio, κ_c, (dotted) as a function of the trap aspect ratio, l, for a purely dipolar condensate with $C_{dd} < 0$.

B. Axially Quartic Potential

We will now investigate the critical trap aspect ratios in the presence of anharmonic potential. Suppose first that anharmonic potential is in the axial direction, $\lambda_\perp = 0$. The selection of the proper form of trial functions is very important in the variational approach. We will choose a trial function which reduces to the ground state solution of the Schrodinger equation in the noninteracting limit. Hence, we employ the trial function up to the first order of λ_z

$$\Psi = \psi_0 \ \exp\left(\frac{-1}{2\sigma^2 d_\perp^2}(r^2 + \frac{z^2}{\kappa^2} + \frac{\lambda_z}{4\kappa^2}(3z^2 + \frac{z^4}{\kappa^2\sigma^2}))\right) \tag{15}$$

where $\psi_0 = \sqrt{\dfrac{N}{\pi^{3/2}\sigma^3 d_\perp^3 \kappa}}\ (1 + \dfrac{9}{32}\lambda_z)$ is the normalization constant. If we evaluate the energy expression, $E^{\lambda_z} = E^{\lambda_z}_{nd} + E^{\lambda_z}_{dd}$, we get

$$\begin{aligned} \frac{E^{\lambda_z}_{dd}}{E_\perp} &= \frac{E^0_{dd}}{E_\perp} - \sqrt{\frac{2}{\pi}} N^\star \lambda_z \frac{G_z(\kappa) + 3\kappa^2(24 - 19\kappa^2)F(\kappa)}{32\kappa(\kappa^2-1)^3} \\ \frac{E^{\lambda_z}_{nd}}{E_\perp} &= \frac{E^0_{nd}}{E_\perp} + \frac{3\lambda_z}{4}\left(\frac{1}{\kappa^2\sigma^2} + \frac{\sigma^2\kappa^2}{l^6}(\sigma^2\kappa^2 - l^2)\right) \end{aligned} \tag{16}$$

where $E_\perp = \frac{1}{2}N\hbar\omega_\perp$, E^0_{dd} and E^0_{nd} are the dipole-dipole and non-dipole energies for purely harmonic trap (9) and $G_z(\kappa) = 14 - 88\kappa^2 + 52\kappa^4 + 7\kappa^6$. The minimums of the variational parameters σ and κ can be calculated from these energy expressions. Then critical aspect ratios can be found. We find that they are decreased with λ_z. For example,

$$l^\star_{\lambda_z=0} = 0.43; \quad l^\star_{\lambda_z=0.05} = 0.41; \quad l^\star_{\lambda_z=0.1} = 0.39. \tag{17}$$

$$l^{\star\star}_{\lambda_z=0} = 2.52, \quad l^{\star\star}_{\lambda_z=0.05} = 2.43 \quad l^{\star\star}_{\lambda_z=0.1} = 2.34\ . \tag{18}$$

The critical number of particles, $N_c^\star$ doesn't change appreciably with λ_z. When $l > l^{\star\star}$, the critical values of the condensate aspect ratio, κ_c, is decreased with λ_z for $C_{dd} > 0$. For example, if $l = 10$ then $\kappa_{c,\lambda_z=0} = 2.60$, $\kappa_{c,\lambda_z=0.05} = 2.49$ and $\kappa_{c,\lambda_z=0.1} = 2.40$. However, $\sigma_c \approx 0.68$ slightly increases with λ_z.

C. Radially Quartic Potential

Suppose now that anharmonic potential is in the radial direction, $\lambda_z = 0$. We will choose our trial function up to the fist order of $\lambda_\perp$

$$\Psi = \psi_0 \exp\left(\frac{-1}{2\sigma^2 d_\perp^2}(r^2 + \frac{z^2}{\kappa^2} + \lambda_\perp(r^2 + \frac{r^4}{4\sigma^2 d_\perp^2}))\right) \tag{19}$$

where $\psi_0 = \sqrt{\dfrac{N}{\pi^{3/2} d_\perp^3 \sigma^3 \kappa}}(1 + \dfrac{3}{4}\lambda_\perp)$ is the normalization constant. The corresponding energy, $E^{\lambda_\perp} = E^{\lambda_\perp}_{nd} + E^{\lambda_\perp}_{dd}$, is given by

$$\begin{aligned} \frac{E^{\lambda_\perp}_{dd}}{E_\perp} &= \frac{E^0_{dd}}{E_\perp} - \sqrt{\frac{2}{\pi}} N^\star \lambda_\perp \frac{G_\perp(\kappa) + 9(32\kappa^2 - 27)F(\kappa)}{96\kappa(\kappa^2-1)^3}\ , \\ \frac{E^{\lambda_\perp}_{nd}}{E_\perp} &= \frac{E^0_{dd}}{E_\perp} + 2\lambda_\perp\left(\sigma^4 - \sigma^2 + \sigma^{-2}\right)\ , \end{aligned} \tag{20}$$

where $G_\perp(\kappa) = 112 + 45\kappa^2 - 258\kappa^4 + 56\kappa^6$.
From the energy expression, we can find the critical trap aspect ratios. The first critical point changes very slowly with $\lambda_\perp$ in contrast to the case of axially anharmonic potential (17).

$$l^\star_{\lambda_\perp=0} = 0.43\ , \quad l^\star_{\lambda_\perp=0.1} = 0.44\ . \tag{21}$$

The second critical point is increased with $\lambda_\perp$. For example,

$$l^{\star\star}_{\lambda_\perp=0} = 2.52, \ l^{\star\star}_{\lambda_\perp=0.05} = 2.56 \ l^{\star\star}_{\lambda_\perp=0.1} = 2.61\ . \tag{22}$$

N_c and σ_c doesn't change appreciably with $\lambda_\perp$. When $l > l^{\star\star}$, κ_c is increased due to the quartic potential. As an example, if $l = 10$, then $\kappa_{c,\lambda_\perp=0} = 2.6$, $\kappa_{c,\lambda_\perp=0.05} = 2.7$ and $\kappa_{c,\lambda_\perp=0.1} = 2.8$.

In this paper, we have shown that there exists some critical trap aspect ratios for a dipolar condensate determining the general structure of the condensate. Classifying the trap as prolate and oblate is not helpful. It was generally believed that critical angular velocity, Ω_c, decreases (increases) due to dipolar interaction in an oblate (a prolate) trap. Here, we have discussed that there exists a critical trap aspect ratio $\gamma_c \approx 2.8$ above (below) which Ω_c is increased (decreased) due to dipolar interaction. We have also shown that the same is true for the precession velocity of an off-axis vortex. Furthermore, we have proved that there are two critical trap aspect ratios for a purely dipolar gas in Gaussian limit. The stability diagram depends crucially on the critical trap ratios. We have shown that the critical points can be changed in the presence of quartic potential.

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